

## RESEARCH PROBLEMS

**Problem 109.** Posed by Peter J. Slater.

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Let  $G$  be a graph with vertex-set  $V = \{1, 2, \dots, n\}$ , edge-set  $E$ , and assume that at each vertex there is a binary switch. Without loss of generality, assume that initially each switch is set to zero. If there exists a walk  $v(1), v(2), \dots, v(2^n - 1)$  in  $G$  with the following properties:

- (i)  $v(i)v(i+1) \in E$  for  $1 \leq i \leq 2^n - 2$ , and
- (ii)  $L(1), L(2), \dots, L(2^n)$  is a list of all binary  $n$ -tuples, where  $L(1)$  is all zeros,  $L(2)$  has a single one in position  $v(1)$ , and in general,  $L(i+1)$  differs from  $L(i)$  only in position  $v(i)$ ;

then  $G$  is called a *Gray codable graph* and  $v(1), v(2), \dots, v(2^n - 1)$  is called a *Gray coding* of  $G$ . Thus, as normally defined, an  $n$ -bit Gray code is a Gray coding of  $K_n$  (usually with the additional requirement that  $L(2^n)$  has a single one so that we can cycle back to  $L(1) = 00 \dots 0$  with a single bit change).

The original question concerning this topic [1] asked if the path  $P_7$  with seven vertices is Gray codable. Using a simple backtracking algorithm and a VAX 750, it was verified not to be Gray codable. In fact, exactly one of the trees on six vertices is not Gray codable, as was also computer verified. Some introductory results are presented in [2].

The problem is to prove that  $P_n$  is not Gray codable for  $n \geq 7$ .

## References

- [1] P.J. Slater, A problem concerning restricted Gray codes, Open problem session, Proc. Tenth Southeastern Conf. Combinatorics, Graph Theory, and Computing (Florida Atlantic Univ.), Congress. Numer. XXIV, Utilitas Math. (Winnipeg, 1979), 918–919.
- [2] P.J. Slater, Graphical Gray codes – An introduction, Technical Report 840301, Univ. Alabama Huntsville.

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Determine which graphs are Gray codable (see the preceding problem for definitions).